

Sliding Mode Control with a State Predictor for Time Delay Control System

T. Ishibashi, K. Kawaguchi, H. Shibasaki, R. Tanaka, T. Murakami, and Y. Ishida

Abstract—In this study, we propose a Sliding Mode Control (SMC) with a state predictor for time delay control systems. Time delay is a factor that causes instability in systems, and SMC is weakened by time delay. Therefore, we use a state predictor with SMC. State predictors are easily designed, and are suitable in combination with SMCs using the state of the plant as a controller. We have successfully realized control using a SMC with a state predictor for the plant with the time delay. In addition, we have confirmed the robustness of the proposed method. The proposed method shows the effective result compared with the conventional method in the simulation.

Index Terms—Time delay, sliding mode control, State predictor.

I. INTRODUCTION

The theory of a Sliding Mode Control (SMC) is based on the concept of changing the structure of a controller in response to the changing state of a system with the aim of obtaining a desired response. A high-speed switching control action is used to switch between different structures, and the trajectory of the system is forced to move along a chosen manifold in the state space, called the switching surface. Therefore, the behavior of the closed loop system is determined by the sliding surface [1]-[3]. In addition, SMC is a nonlinear robust control that attempts to stabilize the system by limiting the state to a switching hyperplane. One of its features is excellent robustness to disturbances and the uncertainties that fulfill the matching condition.

However, according to the principle of SMC, the oscillatory phenomenon called chattering is produced because the existence of a time delay might influence the high-speed switch of the control input, which limits the state to the switching hyperplane [4].

When SMC is applied to an actual system, the state of the system causes oscillations such as high-frequency vibrations and spill-over near the switching hyperplane when chattering occurs, and might lead to the deterioration of the controller.

In this study, we design a state predictor and use SMC to determine the predicted states of the system. A state predictor can be easily designed because we only have to add a predictor to the conventional observer.

This paper is organized as follows: In section II, we describe the design of the system in this study. In section III, the servo system is designed, allowing the state to follow a reference signal. In section IV, we describe the design of the

SMC. In section V, we explain the state predictor. In section VI, we present the entire design as a proposed method. In section VII, we compare the simulation results of the proposed method and conventional method, and we confirm the robustness of SMC with the state predictor. Finally, section VIII concludes the paper.

II. STATE SPACE REPRESENTATION OF CONTROLLED SYSTEM

In this section, the transfer function of the plant $G(s)$ is

$$\begin{aligned} G(s) &= \frac{b}{s(s+a)} e^{-Ls} \\ &= G_p(s) e^{-Ls}. \end{aligned} \quad (2.1)$$

$L \in \mathbb{R}^{1 \times 1}$ is the time delay and $a, b \in \mathbb{R}^{1 \times 1}$ is the plant parameter. $e \in \mathbb{R}^{1 \times 1}$ is an exponential function, and $G_p(s)$ is the system not including the time delay. The transfer function is rewritten in state space representation for the designing of the controller as

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t-L) \\ y(t) = \mathbf{C}\mathbf{x}(t) \end{cases} \quad (2.2)$$

where

$$\begin{aligned} \mathbf{A} &\in \mathbb{R}^{2 \times 2}, \quad \mathbf{B} \in \mathbb{R}^{2 \times 1}, \quad \mathbf{C} \in \mathbb{R}^{1 \times 2}, \\ \mathbf{x}(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in \mathbb{R}^{2 \times 1}, \text{ and } \mathbf{x}(0) = \mathbf{0}. \end{aligned}$$

$y(t) \in \mathbb{R}^{1 \times 1}$ is an output from the plant. In this section, the time delay exists at the input side of the plant, and the plant is a single-input single-output (SISO) system. $\mathbf{x}(t)$ is a state vector, and $u(t) \in \mathbb{R}^{1 \times 1}$ is an input variable.

III. DESIGN OF SERVO SYSTEM

A state space is designed by a spreading system as

$$\begin{cases} \dot{\mathbf{x}}_e(t) = \mathbf{A}_e \mathbf{x}_e(t) + \mathbf{B}_e u(t-L) \\ y_e(t) = \mathbf{C}_e \mathbf{x}_e(t) \end{cases} \quad (3.1)$$

where

$$\begin{aligned} \mathbf{A}_e &= \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & \mathbf{0} \end{bmatrix} \in \mathbb{R}^{3 \times 3}, \quad \mathbf{B}_e = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \in \mathbb{R}^{3 \times 1}, \\ \mathbf{C}_e &= [\mathbf{0} \quad \mathbf{1}] \in \mathbb{R}^{1 \times 3}, \quad \text{and } \mathbf{x}_e(t) = \begin{bmatrix} \mathbf{x}(t) \\ z(t) \end{bmatrix} \in \mathbb{R}^{3 \times 1}; \end{aligned}$$

let

$$z(t) = \int (r(t) - y(t)) dt$$

$$\dot{z}(t) = r(t) - y(t)$$

$r(t) \in \mathbb{R}^{1 \times 1}$ is a reference variable. (3.1) is called the servo-system.

The feedback gain F_1 and servo gain F_2 represent the relative importance of the state variation and control energy consumption, respectively, and are given by the optimal control method as

$$J = \int_0^\infty \{x_e(t)^T Q_e x_e(t) + r_e u(t)^2\} dt \quad (3.2)$$

where $Q_e \in \mathbb{R}^{3 \times 3}$ is a positive definite symmetric matrix, and $r_e \geq 0$. If the optimal state feedback gain F_1 and servo gain F_2 , which minimize the performance index J in (3.2), are rewritten as $F = [F_1 \ F_2]$, F is given by

$$F = -r_e^{-1} B_e^T P_e \quad (3.3)$$

where $F \in \mathbb{R}^{1 \times 3}$, $F_1 \in \mathbb{R}^{1 \times 2}$, and $F_2 \in \mathbb{R}^{1 \times 1}$.

The matrix P_e is the solution of the revised Riccati equation, and is given as

$$P_e A_e + A_e^T P_e - P_e B_e r_e^{-1} B_e^T P_e + Q_e = 0 \quad (3.4)$$

where $P_e \in \mathbb{R}^{3 \times 3}$ [5]-[8].

IV. SLIDING MODE CONTROL

Sliding mode control is expressed as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ \sigma(t) = Sx(t) \end{cases} \quad (4.1)$$

$\sigma(t) \in \mathbb{R}^{1 \times 1}$ is the switching surface and $S \in \mathbb{R}^{1 \times 2}$ is the switching matrix.

A. Design of Switching Matrix

First, to realize control using a sliding mode controller, we are required to design a switching matrix S . S is determined by solving the Riccati equation by employing a method that uses the system's zero-point. The Riccati equation is given as

$$P_s A_s + A_s^T P_s - P_s B_s r_s^{-1} B_s^T P_s + Q_s = 0 \quad (4.1a)$$

where

$$A_s = A + \varepsilon I, \varepsilon \in \mathbb{R}^{1 \times 1} \geq 0 \quad (4.2a)$$

$$S = B^T P_s \quad (4.3a)$$

Let $P_s \in \mathbb{R}^{2 \times 2}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \in \mathbb{R}^{2 \times 2}$, and $r_s \in \mathbb{R}^{1 \times 1}$.

Using this method, $S(sI - A^T + BS)^{-1}B$ becomes strict positive realness, so that its zero is stable [6].

B. Design of Sliding Mode Controller

The control input $u(t)$ consists of a linear input $u_l(t) \in \mathbb{R}^{1 \times 1}$ and nonlinear- input $u_{nl}(t) \in \mathbb{R}^{1 \times 1}$. That is,

$$u(t) = u_l(t) + u_{nl}(t) \quad (4.1b)$$

$u_l(t)$ is designed as a control system of equal value, i.e.,

$$u_l(t) = -(SB)^{-1} SAx(t) \quad (4.2b)$$

$u_{nl}(t)$ is designed as a nonlinear control input as

$$u_{nl}(t) = -K \frac{\sigma(t)}{|\sigma(t)|} \quad (4.3b)$$

$K \in \mathbb{R}^{1 \times 1}$ is a gain of the nonlinear input.

C. Prevention of the Chattering Phenomenon

If a nonlinear control input is designed using (4.3b), a chattering phenomenon occurs. When $\sigma(t) = 0$ for $u_{nl}(t)$, $|\sigma(t)|$ in the denominator becomes 0, and quick switching of the input occurs. This phenomenon is called chattering, which is a high-frequency vibrations. To avoid this problem, (4.3b) is written as

$$u_{nl}(t) = -K \frac{\sigma(t)}{|\sigma(t)| + \eta} \quad (4.1c)$$

where $\eta \in \mathbb{R}^{1 \times 1}$, $\eta > 0$. (4.1c) becomes a smooth function and the chattering phenomenon can be relieved [9]-[14].

V. STATE PREDICTOR

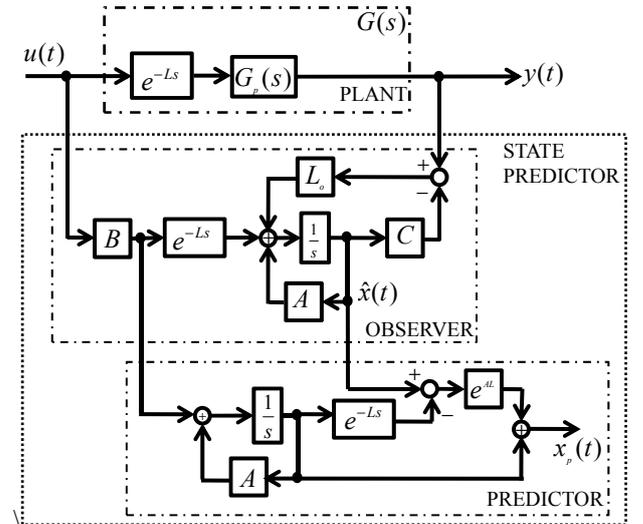


Fig. 1. Block diagram of the state predictor.

Fig. 1 shows the block diagram of the state predictor. Pole placements and optimal control problems for the system with the delay time at the input and output, can be easily settled by using predictive states. A state predictor is designed as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t - L) + L_o(y(t) - \hat{y}(t)) \quad (5.1)$$

$$\hat{y}(t) = C\hat{x}(t) \quad (5.2)$$

$$\dot{x}_p(t) = Ax_p(t) + Bu(t) + L_p(y(t) - \hat{y}(t)) \quad (5.3)$$

where $\hat{x}(t) \in \mathbb{R}^{2 \times 1}$ is the observer state and $x_p(t) \in \mathbb{R}^{2 \times 1}$ is the predicted state. $L_o, L_p \in \mathbb{R}^{2 \times 1}$ are predictor gains and are satisfied with

Next, we compare the proposed method with conventional method. The conventional method consists of the plant, observer, and servo system, but the predictor is not included. Fig. 4 shows the comparison of simulation results of the proposed method with conventional method.

Finally, we have confirmed the robustness of the proposed method. SMC is known as being robust, but the added state predictor is unknown. We rewrite $G(s)$ as $G_e(s)$, and $G_e(s)$ is defined as

$$G_e(s) = \frac{339.6}{s(s+10.78 \times e_m)} e^{-0.04s}, \quad (7.6)$$

$$e_m = 1.5,$$

where $e_m \in \mathbb{R}^{1 \times 1}$ is the modeling error. The state predictor parameters (A, B, C, L) are the same as (7.2). Fig.5 shows the robustness of the proposed method.

VIII. CONCLUSION

In this study, we have controlled a plant with time delay using SMC with a state predictor. Simulation results show that the proposed method is improved when compared with the conventional method, and we have confirmed that the proposed method is effective for an input side disturbance.

Furthermore, we have confirmed the robustness, which is one of the characteristics of SMC when we add a state predictor.

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