

# The Ideal Map Projection for Sea Ports

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**Abstract**—The modern technologies and developments in computers and Global Positioning System (GPS) as well as Geographic Information System (GIS), become very important in present time in mapping and sea navigation. Traditional map projection systems are not suitable for modern technologies, because they haven't high accuracy in determining the position of features, also when calculating the distances compared by Indirect Problems of Geodesy (I.P.G) it have bad results.

This paper presents new method for map projection use in sea navigation specially in sea narrow paths were the risk of ship crash become very high. The map projection by harmonic equations method was used. The results show that the error in short distances was between 0.0 and 30.0 cm compared with 20.0 cm and 450.0 cm error when UTM system was used. Local system method by harmonic equations shows better results than other methods.

**Index Terms**—Map projection, GPS, GIS, Harmonic equations, UTM, Geodesy

## I. INTRODUCTION

The modern technologies and developments in computers and Global Positioning System (GPS) as well as Geographic Information System (GIS), become very important in present time in mapping and sea navigation. Traditional map projection systems are not suitable for modern technologies, because they haven't high accuracy in determining the position of features, also when calculating the distances compared by Indirect Problems of Geodesy (I.P.G) it have bad results [1].

In 1998 Belarusian Professor “doctor science” Vladimir Badshevalov develop a new theory for map projections “the united map projections geodetic” (Lambert , Mercator , Russell ,Lagrange and compound projection) [2], while Dr. Akresh M.S 2009, 2011, 2012 find the general law of the inverse of algorithms, direct algorithms in Russell projection for the theory Prof. Vladimir Badshevalov (harmonic equations) [3].

## II. METHODOLOGY

The research methodology uses map projection by harmonic equations using the following steps:

Constructing coordinates system using geographic coordinates for boundaries of U.K territory, as well as standard parallel and central meridian for zone in U.K (11×14 degree) see (Fig. 1).

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$$\begin{aligned} \phi_s &= 49^\circ 55' 00'' \text{ N} & \lambda_w &= 11^\circ 00' 00'' \text{ W} \\ \phi_N &= 61^\circ 00' 00'' \text{ N} & \lambda_E &= 02^\circ 30' 00'' \text{ E} \\ \phi_p &= 55^\circ 30' 00'' & NL_c &= 4^\circ 30' 00'' \text{ W} \end{aligned}$$

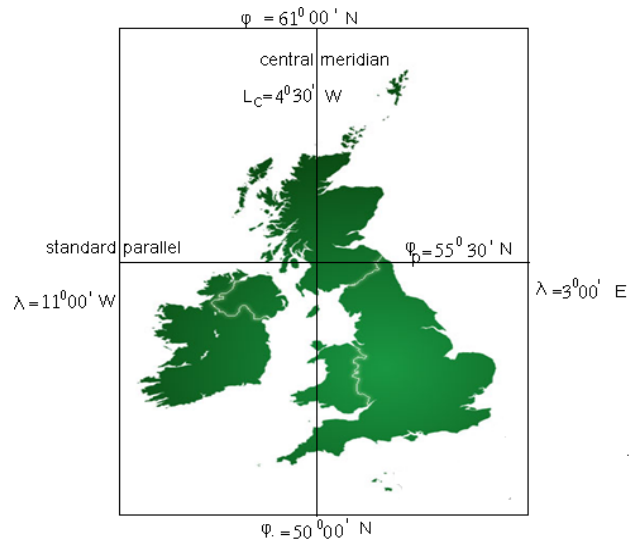


Fig. 1. U.K in one zone projection

Scale factor for the zone can be obtained from the following equation:

$$m = 1 + \frac{y_{\max}^2}{2R^2}, \quad y_{\max} = N \cos \beta_s \cdot \frac{\frac{n}{180} \pi}{2}; \quad m_0 = \frac{2}{1+m} \quad (1)$$

where:

- $n$ - The width of zone in degrees (here 11X14);
- $m$ - Normal scale factor;
- $m_0$ - Ideal scale factor.

Here uses elements of ellipsoid WGS-84

$$m = 1.001762874$$

$$m_0 = 0.99911934$$

The ideal scale factor for Mercator projection gives less distortion in edges, while normal scale factor has less distortion in center zone.

All coordinate systems in map projections can be created by one of these two methods: direct method and indirect method [2], [3].

### A. Direct Method

uses geographic coordinate transformation  $(\phi, \lambda)$  to rectangular coordinate in  $(x, y)$ ; the fundamentals of equations are as following [2]-[4]:

$$\left. \begin{aligned} x &= X_0 + C_1 P_1 + C_2 P_2 + C_3 P_3 + \dots \\ y &= Y_0 + C_1 Q_1 + C_2 Q_2 + C_3 Q_3 + \dots \end{aligned} \right\} \quad (2)$$

where:  $X_0, Y_0$  = initials coordinates systems for zone

projection;

$C_j$ = coefficients expansion of projection by direct method;  
 $P_j, Q_j$ = elements of harmonic multinomial equations apply to Laplace equations.

An initial coordinates systems for zone projection; can be found from meridian arc, and parallels ellipsoid[2]-[4].

$$X_0 = n_1 B_p - n_2 \sin 2B_p + n_3 \sin 4B_p - n_4 \sin 6B_p + \dots \quad (3)$$

$$\begin{aligned} P_j &= P_{j-1} P_1 - Q_{j-1} Q_1, & P_0 &= 1 \\ Q_j &= P_{j-1} Q_1 + Q_{j-1} P_1, & Q_0 &= 0 \end{aligned} \quad (4)$$

where:  $P_j$ = different values between latitudes;  
 $Q_j$ = different values between longitudes.

The different values between latitudes may calculated from  $q$  (isometric latitude), and difference between longitudes begin from  $L_c$  (center meridian) to get meridian; isometric latitude value can be computed from following equation:

$$q = \ln \sqrt{\left( \frac{1 + \sin B}{1 - \sin B} \right) \left( \frac{1 - e \sin B}{1 + e \sin B} \right)^e} \quad (5)$$

The difference between map projections by harmonic equations (Mercator, Lambert and Russell, Lagrange and compound projection) only in coefficients, where any hair has special coefficients; here we will use Mercator projection for U.K, direct coefficients are as following [3], [5]:

$$\begin{aligned} C_1 &= \frac{m_0 \cdot c \cdot \cos B_0}{V}, & C_2 &= -\frac{C_1 \cdot \sin B_0}{2}, \\ C_3 &= \frac{C_1 \cdot \cos^2 B_0}{6} (\tan^2 B_0 - V^2), \end{aligned} \quad (6)$$

$$C_{12} = \dots$$

For all coefficients see references [4]

### B. Indirect Method

Uses rectangular coordinate transformation ( $x, y$ ) to geographic coordinate ( $\varphi, \lambda$ ); the Fundamentals equations are as following [3], [6]:

$$q = q_0 + \sum_{j=1}^n C_j P_j \quad (7)$$

$$L = L_0 + \sum_{j=1}^n C_j Q_j$$

$$\begin{aligned} P_j &= P_{j-1} P_1 - Q_{j-1} Q_1, & P_0 &= 1 \\ Q_j &= P_{j-1} Q_1 + Q_{j-1} P_1, & Q_0 &= 0 \end{aligned} \quad (8)$$

Indirect coefficients for all projections can be computed from following equation:

$$\begin{aligned} C'_1 &= \frac{1}{C_1}, & C'_2 &= -\frac{C_2}{C_1^3}, & C'_3 &= \frac{1}{C_1^5} (2C_2^2 - C_1 C_3), \\ C'_4 &= \frac{1}{C_1^7} (-5C_2^3 - C_1^2 C_4 + 5C_1 C_2 C_3), \end{aligned} \quad (9)$$

$$C_j = \dots$$

Geographic latitude using iteration value by following equation

$$B = 2 \arctan \left[ \sqrt{\left( \frac{1 + e \sin B}{1 - e \sin B} \right)^e} * \exp(q) \right] - \frac{\pi}{2} \quad (10)$$

Hereis some examples for estimating general Mercator projection by harmonic equation; the transformation of geographic coordinates to rectangular coordinates system, and vice versa results are shown in (Table I).

TABLE I: TRANSFORMATION GEOGRAPHIC COORDINATES IN RECTANGULAR COORDINATES AND INVERSE

Transformation of geographic coordinates to rectangular coordinates					
	$\varphi$	$\lambda$	X	Y	f.sc. point $m_0$
1	50°00'00.000 0"N	10°30'00.000 0"W	5558648.5 959	429658.80 20	1.00138 847
2	50°00'00.000 0"N	11°30'00.000 0"W	5564893.8 052	501210.13 85	1.00220 751
3	61°00'00.000 0"N	10°30'00.000 0"W	6779813.6 469	324044.26 51	1.00040 673
4	61°00'00.000 0"N	11°30'00.000 0"W	6785182.3 265	377918.60 84	1.00087 041
Transformation of rectangular coordinates to geographic coordinates					
	X	Y	$\varphi$	$\lambda$	
1	5558648.595 9	429658.8020	50°00'00.0000"N	10°30'00.0000"W	
2	5564893.805 2	501210.1385	49°59'59.9999"N	11°30'00.0000"W	
3	6779813.646 9	324044.2651	61°00'00.0000"N	10°30'00.0000"W	
4	6785182.326 5	377918.6084	61°00'00.0000"N	11°29'59.9998"W	

The results show that there is no error (1mm) in the case of transforming coordinates to geographic coordinates.

### III. LOCAL SYSTEM

The local system gives a good results in the present time, the local system gives rectangular coordinate system for each city within high accuracy in distances measurements “without Sampson correction method”, and it’s equal distances measured by indirect geodetic problems.

These problems solved by new methodologies in geodetic projections method “multinomial harmonic equations” by Laplace equations, as well as has the easy way to make rectangular transformation between local system and general system as following:

$$dX = \frac{m_0}{m'_0} dx, \quad dY = \frac{m_0}{m'_0} dy, \quad X = X_0 + dX,$$

$$Y = Y_0 + dY$$

where:  $m_0$  : ideal scale factor for main projection;

$m'_0$  : ideal scale factor for local projection;

$X_0, Y_0$ : initials coordinates systems for main projection;

$dx, dy$  : coordinates system for local projection;

$dX, dY$  : coordinates system for main projection.

The local system has overlapped area for cities ( $20 \times 20 \text{ km}^2$ ), good results can be obtained compared by traditional Mercator projection (UTM); here, local system was used for two sea ports (Landon sea port and Liverpool sea port).

Table II shows the geographical coordinates of London city at local system. Point "E" is the central point of the system where the scale factor equal to 1.

Table III shows the coordinates of the points in Table II computed by three different projection methods; 1- Local system by harmonic equations Mercator projection, 2- UTM projection, and 3- Mercator projection by harmonic equations.

The local system by harmonic equations gives a good results compared with the two other methods. The maximum error in the area was  $\pm 45.82 \text{ sq. m.}$  in the local system compared with  $\pm 1865.06 \text{ sq. m.}$  and  $13032.06 \text{ sq. m.}$  in the other two methods.

TABLE II: POINTS IN LONDON SEA PORT.

points	$\phi$	$\lambda$
A	51°31'40.00" N	00°39'16.88" E
B	51°29'45.00" N	00°38'56.81" E
C	51°29'06.12" N	00°44'32.83" E
Center point for local system		
E	51°29'30.00" N	00°40'30.00" E

TABLE IV: POINTS IN LIVERPOOL SEA PORT.

points	$\Phi$	$\lambda$
A	53°27'02.68" N	3°01'54.31" W
B	53°24'42.56" N	3°11'19.85" W
C	53°29'00.00" N	3°11'55.00" W
Center point for local system		
E	53°27'00.00" N	3°03'00.00" W

(Table IV) shows the geographical coordinates of Liverpool city at local system. Point "E" is the central point of the system where the scale factor equal to 1. compared with the other two methods.

(Table V) shows the coordinates of the points in (Table IV) computed by three different projection methods; 1- Local system by harmonic equations Mercator projection, 2- UTM projection, and 3- Mercator projection by harmonic equations.

The local system by harmonic equations gives a good results compared with the two other methods. The maximum error in the area was  $\pm 1261.7 \text{ sq. m.}$  in the local system compared with  $\pm 34145.5 \text{ sq. m.}$  and  $67095.9 \text{ sq. m.}$  in the other two methods.

TABLE III: DATA ANALYSIS OF LONDON SEA PORT.

Local system for port Landon $X_0=2932230.4925 \text{ m, } Y_0=0 \text{ m, } m_0=0.99841771$			
	A	B	C
X - meters	5724091.9495	5720519.4995	5719781.0688
Y - meters	357051.6219	356915.7519	363467.6276
$m'_0$ point	0.99998511	0.99998393	1.00004198
S - form X,Y m	$S_{A-B}=3575.033 \text{ m}$	$S_{B-C}=6593.357 \text{ m}$	$S_{C-A}=7729.736 \text{ m}$
S -by Sampson m	$S_{A-B}=3575.088 \text{ m}$	$S_{B-C}=6593.272 \text{ m}$	$S_{C-A}=7729.631 \text{ m}$
S- from I.P.G m	$S_{A-B}=3575.088 \text{ m}$	$S_{B-C}=6593.272 \text{ m}$	$S_{C-A}=7729.632 \text{ m}$
Area by X,Y= $11753290.36 \text{ m}^2 \pm 45.82 \text{ m}^2$		Area by Sam= $11753336.18 \text{ m}^2 \pm 00.00 \text{ m}^2$	Area by I.P.G= $11753336.18 \text{ m}^2 \pm 00.00 \text{ m}^2$
projection UTM Mercator $m_0=0.9996$			
	A	B	C
X - meters	5711125.882	5707586.312	5706181.425
Y - meters	337314.741	336813.933	343255.212
$m'_0$ point	0.99993468	0.99992668	0.99990140
S - form X,Y	$S_{A-B}=3574.823 \text{ m}$	$S_{B-C}=6592.707 \text{ m}$	$S_{C-A}=7728.961 \text{ m}$
S -by Sampson	$S_{A-B}=3575.071 \text{ m}$	$S_{B-C}=6593.274 \text{ m}$	$S_{C-A}=7729.594 \text{ m}$
S- from I.P.G	$S_{A-B}=3575.088 \text{ m}$	$S_{B-C}=6593.272 \text{ m}$	$S_{C-A}=7729.632 \text{ m}$
Area by X,Y= $11751471.12 \text{ m}^2 \pm 1865.06 \text{ m}^2$		Area by Sam= $11753297.24 \text{ m}^2 \pm 38.90 \text{ m}^2$	Area by I.P.G= $11753336.18 \text{ m}^2 \pm 00.00 \text{ m}^2$
General projection in Mercator "harmonic" $X_0=2932230.4925 \text{ m, } Y_0=0 \text{ m, } m_0=0.99911934$			
	A	B	C
X - meters	5723790.6121	5720215.6516	5719476.7019
Y - meters	357302.5371	357166.5716	363723.0515
$m'_0$ point	1.00068784	1.00068666	1.00074475
S - form X,Y	$S_{A-B}=3576.546 \text{ m}$	$S_{B-C}=6597.990 \text{ m}$	$S_{C-A}=7735.1686 \text{ m}$
S -by Sampson	$S_{A-B}=3575.088 \text{ m}$	$S_{B-C}=6593.268 \text{ m}$	$S_{C-A}=7729.628 \text{ m}$
S- from I.P.G	$S_{A-B}=3575.088 \text{ m}$	$S_{B-C}=6593.272 \text{ m}$	$S_{C-A}=7729.632 \text{ m}$
Area by X,Y= $11766369.02 \text{ m}^2 \pm 13032.8 \text{ m}^2$		Area by Sam= $11753311.05 \text{ m}^2 \pm 25.13 \text{ m}^2$	Area by I.P.G= $11753336.18 \text{ m}^2 \pm 00.00 \text{ m}^2$

TABLE V: DATA ANALYSIS OF LIVERPOOL SEA PORT.

Local system for port Liverpool $X_0=2932230.4925\text{ m}$ , $Y_0=0\text{ m}$ , $m_0=0.999886173$			
	A	B	C
X - meters	5925813.9066	5921279.5444	5929224.9642
Y - meters	97530.186	87175.1618	86380.8039
$m'_0$ point	1.00000287	0.99997941	0.99997772
S - form X,Y m	$S_{A-B}=11304.290\text{m}$	$S_{B-C}=7985.030\text{m}$	$S_{C-A}=11659.504\text{m}$
S -by Sampson m	$S_{A-B}=11304.390\text{ m}$	$S_{B-C}=7985.201\text{m}$	$S_{C-A}=11659.620\text{m}$
S- from I.P.G m	$S_{A-B}=11304.392\text{m}$	$S_{B-C}=7985.201\text{m}$	$S_{C-A}=11659.620\text{m}$
Area by X,Y= $42938462.0\text{m}^2 \pm 1262.7\text{m}^2$		Area by Sam= $42939724.7\text{m}^2 \pm 00.00\text{ m}^2$	Area by I.P.G= $42939724.7\text{m}^2 \pm 00.00\text{ m}^2$
projection UTM Mercator $m_0=0.9996$			
	A	B	C
X - meters	5922414.509	5918100.558	5926057.929
Y - meters	497891.318	487447.320	486820.468
$m'_0$ point	0.99960005	0.99960193	0.99960213
S - form X,Y	$S_{A-B}=11299.879\text{m}$	$S_{B-C}=7982.043\text{m}$	$S_{C-A}=11654.966\text{m}$
S -by Sampson	$S_{A-B}=11304.388\text{m}$	$S_{B-C}=7985.220\text{m}$	$S_{C-A}=11659.615\text{m}$
S- from I.P.G	$S_{A-B}=11304.392\text{m}$	$S_{B-C}=7985.201\text{m}$	$S_{C-A}=11659.620\text{m}$
Area by X,Y= $42905579.2\text{m}^2 \pm 34145.5\text{m}^2$		Area by Sam= $42939871.2\text{m}^2 \pm 146.5\text{m}^2$	Area by I.P.G= $42939724.7\text{m}^2 \pm 00.00\text{ m}^2$
General projection in Mercator "harmonic" $X_0=2932230.4925\text{m}$ , $Y_0=0\text{ m}$ , $m_0=0.99911934$			
	A	B	C
X - meters	5925988.0592	5921457.1745	5929396.5008
Y - meters	97455.3888	87108.3054	86314.5568
$m'_0$ point	0.99923595	0.9992125	0.99921081
S - form X,Y	$S_{A-B}=11295.621\text{m}$	$S_{B-C}=7978.906\text{m}$	$S_{C-A}=11650.563\text{m}$
S -by Sampson	$S_{A-B}=11304.384\text{m}$	$S_{B-C}=7985.200\text{m}$	$S_{C-A}=11659.611\text{m}$
S- from I.P.G	$S_{A-B}=11304.392\text{m}$	$S_{B-C}=7985.201\text{m}$	$S_{C-A}=11659.620\text{m}$
Area by X,Y= $42872628.8\text{m}^2 \pm 67095.9\text{m}^2$		Area by Sam= $42939714.2\text{m}^2 \pm 10.5\text{ m}^2$	Area by I.P.G= $42939724.7\text{m}^2 \pm 00.00\text{ m}^2$

IV. CONCLUSION

A study has been done to obtain the ideal projection for sea ports. From the results obtained, it can be conclude that the local system is better than the traditional projection.

Local systems used in the projections of harmonic equations give good results for the distances and areas measurement on maps without "Simpson modified".The results show that the error in short distances was between 0.0 and 30.0 cm compared with 20.0 cm and 450.0 cm error when UTM system was used. Local system method by harmonic equations shows better results than other methods.

Local coordinate system is better than the UTM coordinate system in certain places specially for locating the environmental pollution like oil spills in sea ports and coastal areas.

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